Soft Isometry and Completions of Soft metric space

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***Abstract*—. In this paper we define soft isometry and discuss some results on soft isometric spaces. We also define completion of soft metric and prove that there exists a unique completion of every soft metric space up to isometry.**

Keywords— Soft set, Soft point, Soft metric, Soft open ball, Soft bounded set.

Introduction (*Heading 1*)

Many times problems in economics, engineering environmental sciences, medical sciences, etc involve uncertainties, imprecision and vagueness and these problems cannot be modeled using classical mathematical tools. To overcome these problems, a number of mathematical theories such as probability theory, interval mathematics theory, fuzzy set theory, vague set theory, etc. formulated to solve such problems, and these models were successful but with some limitations [4]. The major reason for the difficulties arising with the above mentioned theories is due to the inadequacies of their parameterization tools [4]. In order to overcome these difficulties, in the year 1990, Molodtsov [4] introduced completely new mathematical tool called soft set theory. In recent years many researches like Feng et al. [2], Maji et al.[3] Shabir and Naz [5], Sujao Das and Samanta [6]-[7], B. Surendranath Reddy and Sayed Jalil [1] etc. have contributed to the development soft set theory. In this paper section 1 is about preliminaries. In section 2, we define soft isometry and derive some properties of it. In section 3, we define completion of a soft metric space and prove that every soft metric has unique completion up to isometry.

I. PRELIMINARIES

In this section we recall basic definitions and results about soft sets.

Definition 1.1.[4] Let U be the universe and E be the set of parameters. Let $P(U)$ denote the power set of U and A be non empty subset of E. A pair (F, A) is called a soft set over U where F is given by $F : A \rightarrow P(U)$.

In other words, the soft set is parameterized family of subsets of the set U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε-elements of the set (F, A) or as the set of ε-approximate elements of the soft set (F, A) is given as consisting of collection of approximation:
$(F, A) = \{ F(\varepsilon) | \ \varepsilon \in A \}$.

Definition 1.2.[2] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is soft subset of (G, B) if

1. $A \subseteq B$ and
2 $\forall, e \in A, \quad F(e) \subseteq G(e)$.

And it is denoted by $(F, A) \tilde{\subseteq} (G, B)$.

Definition 1.3.[3] Two soft sets (F, A) and (G, B) over a common universe U, are said to be equal if (F, A) is soft subset of (G, B) and (G, B) is soft subset of (F, A).

Definition 1.4.[4] The union of two soft sets (F, A) and (G, B) over the common universe U, is the soft set (H, C), where $C = A \cup B$ and $\forall \ \ e \in C$,

$$H(e) = \begin{cases} F(e), \ if e \\ G(e), \ if e \\ F(e) \cup G(e), \ i \end{cases}$$

And it is denoted by $(F, A) \tilde{\cup} (G, B) = (H, C)$

Definition 1.5.[3] The intersection of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C) where $C = A \cap B$ and $\forall \ e \in C$, $H(e) = F(e) \cap G(e)$ and is denoted by $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Definition 1.6[6] The difference (H, E) of two soft sets (F, E) and (G, E) over U is denoted by $(H, E) = (F, E) \setminus (G, E)$ is defined as
$H(e) = F(e) \setminus G(e), \qquad \forall e \in E.$

Definition 1.7[2] The complement of a soft set (F, A) over U is denoted by $(F, A)^{C} = (F^{C}, A)$, and is defined as

$(F,A)^c=(F^c,A),$ where $F^c:A\to P(U)$ is a mapping given by $F^c(\lambda)=U-F(\lambda)=F(\lambda)^c,\ \forall,\lambda\in A$
i.e. $(F,A)^c=\{F(e_i)^c,\forall,e_i\in A\}.$

Definition 1.8[3] A soft set (F,E) over U is said to be a null soft set denoted by $\tilde{\phi}$ if $F(e)=\phi\quad\forall e\in E.$

Definition 1.9[3] A soft sets (F,E) over U is said to be absolute soft set denoted by $\tilde{U}$ if $F(e)=U\quad\forall e\in E$.

Definition 1.10[8] Let X be a non empty set and E be a non empty parameter set then the function $\varepsilon:E\to X$ is said to be soft element of .
A soft element ε is said to belongs to a soft set (F,A) of X if $\varepsilon(e)\in A(e)$, $\forall e\in A$ and is denoted by $\varepsilon\tilde{\in}(F,A)$.

Definition 1.11[8] Let $\mathbb{R}$ be the set of real numbers and $B(\mathbb{R})$ the collection of all non empty bounded subsets of $\mathbb{R}$ and A be a set of parameters then the mapping $F:A\to B(\mathbb{R})$ is called a soft real set. It is denoted by (F,A). In particular, if (F,A) is singleton soft set then identifying (F,A) with the corresponding soft element, it will be called a soft real number.
We denote soft real numbers by $\tilde{r},\tilde{s},\tilde{t}$ and $\bar{r},\bar{s},\bar{t}$ will denote a particular type of soft numbers such that $\bar{r}(\lambda)=r,\forall\lambda\in A$ etc. For example $\bar{0}(\lambda)=0$ and $\bar{1}(\lambda)=1,\forall\lambda\in A$

Definition 1.12[7] A soft set (F,A) over X is said to be a soft point if there is exactly one $\lambda\in A$, such that $P(\lambda)=x$, for some $x\in X$ and $P(\mu)=\phi,\forall\mu\in A\setminus\{\lambda\}$. It is denoted by $P_\lambda^{\ x}$.

Definition 1.13[7] A soft point $P_\lambda^{\ x}$ is said to belong to a soft set (F,A) if $\lambda\in A$ and $P(\lambda)=\{x\}\subset F(\lambda)$ and we write $P_\lambda^{\ x}\tilde{\in}(P,A)$.

Dentition 1.14[1] Two soft points $P_\lambda^{\ x}$ and $P_\mu^{\ y}$ are said to be equal if $\lambda=\mu$ and $P(\lambda)=P(\mu)$ i.e. $x=y$ .Thus $P^x{}_\lambda\neq P^y{}_\mu\Leftrightarrow x\neq y$ or $\lambda\neq\mu$.

Let X be an initial universal set and A be a non empty set of parameters. Let $\tilde{X}$ be the absolute soft set. Let $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$. Let $\mathbb{R}(A^*)$ denote the set of all non negative soft real numbers.

Definition 1.15[7] A mapping $\tilde{d}:SP(\tilde{X})\times SP(\tilde{X})\to\mathbb{R}(A^*)$ is said to be a soft metric on the soft set $\tilde{X}$ if
(i) $\tilde{d}(P_\lambda^{\ x},P_\mu^{\ y})\tilde{\geq}\tilde{0},\forall P_\lambda^{\ x},P_\mu^{\ y}\in\tilde{X}$
(ii) $\tilde{d}(P_\lambda^{\ x},P_\mu^{\ y})=\tilde{0}$, iff $P_\lambda^{\ x}=P_\mu^{\ y}$
(iii) $\tilde{d}(P_\lambda^{\ x},P_\mu^{\ y})=\tilde{d}(P_\mu^{\ y},P_\lambda^{\ x}),\ \forall P_\lambda^{\ x},P_\mu^{\ y}\in\tilde{X}$
(iv) $\tilde{d}(P_\lambda^{\ x},P_\mu^{\ y})\tilde{\leq}\tilde{d}(P_\lambda^{\ x},P_\gamma^{\ z})+\tilde{d}(P_\gamma^{\ z},P_\mu^{\ y})\ \forall P_\lambda^{\ x},P_\mu^{\ y}\in\tilde{X}$

The soft set $\tilde{X}$ with the soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X},\tilde{d},E)$ or $(\tilde{X},\tilde{d})$

Example 1.16[7] Let $\tilde{X}$ be a non empty set and E be the non empty set of parameters. Let $\tilde{X}$ be the absolute soft set
i.e $F(\lambda)=X,\forall\lambda\in A,$ where $(F,A)=\tilde{X}$.
Define $\tilde{d}:SP(\tilde{X})\times SP(\tilde{X})\to\mathbb{R}(A^*)$ by,

$$\tilde{d}:(P_\lambda^{\ x},P_\mu^{\ y})=\begin{cases}\tilde{0}, & if\ P_\lambda^{\ x}=P_\mu^{\ y}\\ \tilde{1}, & if\ P_\lambda^{\ x}\neq P_\mu^{\ y}\end{cases}$$

Then $\tilde{d}$ is a soft metric and is called soft discrete metric and $(\tilde{X},\tilde{d},E)$ is called soft discrete metric space.

Definition 1.17[6] Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and $\dot{r}$ be a non negative soft real number. Then the set $B(P_{\lambda,}^{\ x}\tilde{r})=\{P_\mu^{\ y}\in SP(\tilde{X})\,|\,\tilde{d}(P_\lambda^{\ x}P_\mu^{\ y})<\tilde{r}\}$ is called soft open ball with center $P_\lambda^{\ x}$ and of radius $\dot{r}$.

Definition 1.18[6] Let $(\tilde{X},\tilde{d},E)$ be soft metric space (Y,A) be a non null soft subset of $\tilde{X}$ Then (Y,A) is said to be soft open in $(\tilde{X},d)$ if and only if $\forall P_\lambda^{\ x}\tilde{\in}(Y,A)$ there exist a soft real number $\tilde{r}\tilde{>}\tilde{0}$ such that $B(P_\lambda^{\ x},\tilde{r})\tilde{\subset}(Y,A)$.

Definition 1.19[6] Let $(\tilde{X},\tilde{d},E)$ be soft metric space (Y,A) be a non null soft subset of $\tilde{X}$ then the diameter of (Y,A) is denoted by $diam((Y,A))$ and for any $\gamma\in A,$ it is defined as,
$diam((Y,A))(\gamma)=\sup\{\tilde{d}(P_\lambda^{\ x},P_\mu^{\ y})(\gamma):P_\lambda^{\ x},P_\mu^{\ y}\tilde{\in}(Y,A)\}$

Definition 1.20[1] Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and (Y,A) be a non null soft subset of $\tilde{X}$ .Then we say that (Y,A) is soft bounded if there exists $P_\lambda^{\ x}\tilde{\in}\tilde{X}$ and a soft real number $\tilde{\varepsilon}\tilde{>}\tilde{0},$ such that $(Y,A)\tilde{\subset}B(P_\lambda^{\ x},\tilde{\varepsilon})$.

Definition 1.21[6] A soft metric space $(\tilde{X},\tilde{d},E)$ is said to be soft separable if there exists a countable soft subset (Y,A) of $\tilde{X}$ such that $\overline{(Y,A)}=\tilde{X}$.

Definition 1.22 Let $(\tilde{X},\tilde{d}_1,E_1)$ and $(\tilde{Y},\tilde{d}_2,E_2)$ be two soft metric spaces. Then the mapping $(f,\phi):(\tilde{X},\tilde{d}_1,E_1)\to(\tilde{Y},\tilde{d}_2,E_2)$ is called soft mapping where $f:X\to Y$ and $\phi:E_1\to E_2$ are two mappings.

Definition 1.23 Let $(\tilde{X},\tilde{d}_1,E_1)$ and $(\tilde{Y},\tilde{d}_2,E_2)$ be two soft metric spaces. The soft mapping $(f,\phi):(\tilde{X},\tilde{d}_1,E_1)\to(\tilde{Y},\tilde{d}_2,E_2)$ is said to be soft continuous at the soft point $P_\lambda^{\ x}\tilde{\in}SP(\tilde{X})$, if for every $\tilde{\varepsilon}\tilde{>}\tilde{0}$, there exist a $\tilde{\delta}\tilde{>}\tilde{0}$, such that for any soft points $P_\lambda^{\ x},P_\mu^{\ y}\in\tilde{X}$, whenever $\tilde{d}_1(P_\lambda^{\ x},P_\mu^{\ y})\tilde{\le}\tilde{\delta}\Rightarrow\tilde{d}_2((f,\phi)P_\lambda^{\ x},(f,\phi)P_\mu^{\ y}))\tilde{\le}\tilde{\varepsilon}$

Definition 1.24 Let $(\tilde{X},\tilde{d}_1,E_1)$ and $(\tilde{Y},\tilde{d}_2,E_2)$ be two soft metric spaces. The soft mapping $(f,\phi):(\tilde{X},\tilde{d}_1,E_1)\to(\tilde{Y},\tilde{d}_2,E_2)$ is said to be soft uniformly continuous mapping if for every $\tilde{\varepsilon}\tilde{>}\tilde{0}$, there exist a $\tilde{\delta}\tilde{>}\tilde{0}$, ($\tilde{\delta}$ depends only on $\tilde{\varepsilon}$) such that
$\tilde{d}_1(P_\lambda^{\ x},p_\mu^{\ y})\tilde{\le}\tilde{\delta}\Rightarrow\tilde{d}_2((f,\phi)P_\lambda^{\ x},(f,\phi)P_\mu^{\ y}))\tilde{\le}\tilde{\varepsilon}$

II. Soft Isometry

Definition 2.1. Let $(\tilde{X},\tilde{d}_1,E_1)$ and $(\tilde{Y},\tilde{d}_2,E_2)$ are said to be soft isometric if there exists a soft mapping $(f,\phi):(\tilde{X},\tilde{d}_1,E_1)\to(\tilde{Y},\tilde{d}_2,E_2)$ satisfying

$$\tilde{d}_2((f,\phi)(P_\lambda^{\ x}),(f,\phi)(P_\mu^{\ y}))=\tilde{d}_1(P_\lambda^{\ x},P_\mu^{\ y})\forall P_\lambda^{\ x},P_\mu^{\ y}\in\tilde{X}.\ .$$

The soft map (f,ϕ) is called a soft isometry.
We can observe that soft isometry map is bijective.

Theorem 2.2. Let $(\tilde{X},\tilde{d}_1,E_1)$ be a soft bounded metric space, and $(\tilde{Y},\tilde{d}_2,E_2)$ be another soft metric space. Let $(f,\phi):(\tilde{X},\tilde{d}_1,E_1)\to(\tilde{Y},\tilde{d}_2,E_2)$ is a soft isometry then $(\tilde{Y},\tilde{d}_2,E_2)$ is soft bounded.

Poof: Since (f,ϕ) is an isometry,
$\tilde{d}_2((f,\phi)(P_\lambda^{\ x}),(f,\phi)(P_\mu^{\ y}))=\tilde{d}_1(P_\lambda^{\ x},P_\mu^{\ y})\forall P_\lambda^{\ x},P_\mu^{\ y}\in\tilde{X}.$
Also it is given that $(\tilde{X},\tilde{d}_1,E_1)$ be a soft bounded metric space.
$\Rightarrow\tilde{d}_1(P_\lambda^{\ x},P_\mu^{\ y})\tilde{<}\tilde{k}$
$\Rightarrow\tilde{d}_2((f,\phi)(P_\lambda^{\ x}),(f,\phi)(P_\mu^{\ y}))\tilde{<}\tilde{k}$
Therefore, $(\tilde{Y},\tilde{d}_2,E_2)$ is soft bounded.

Theorem 2.3. Soft separablity is preserved under isometry.

Proof: Let $(\tilde{X},\tilde{d}_1,E_1)$ be a soft separable metric space, and $(\tilde{Y},\tilde{d}_2,E_2)$ be another soft metric space. Let $(f,\phi):(\tilde{X},\tilde{d}_1,E_1)\to(\tilde{Y},\tilde{d}_2,E_2)$ is a soft isometry.
Assume that (H,A) is countable and dense set in $\tilde{X}$. Since (f,ϕ) is isometry i.e. injective and surjective, $(f,\phi)(H,A)$ is countable. Let $P_\mu^{\ y}$ be a soft point in $(\tilde{Y},E_2)-(f,\varnothing)(H,A)$
Then there exist a soft point $P_\lambda^{\ x}\in\tilde{X}$ such that, $(f,\phi)(P_\lambda^{\ x})=P_\mu^{\ y}$.
Let $\tilde{\varepsilon}\tilde{>}\tilde{0}$ be a soft real number then there exist a soft point $P_{\lambda'}^{\ x'}$ in (H,A) such that $P_{\lambda'}^{\ x'}$ is in $B_{\tilde{d}_1}(P_\lambda^{\ x},\tilde{\varepsilon})$.
Since $(f,\phi)(P_{\lambda'}^{\ x'})\tilde{\in}(f,\phi)(H,A)$ and

$$\tilde{d}_2((f,\phi)(P_{\lambda'}^{\ x'}),P_\mu^{\ y})=\tilde{d}_2((f,\phi)(P_{\lambda'}^{\ x'}),(f,\phi)(P_\lambda^{\ x}))$$
$$=\tilde{d}_1(P_{\lambda'}^{\ x'},P_\lambda^{\ x})\tilde{<}\tilde{\varepsilon}$$

$\Rightarrow P_\mu^{\ y}$ is a limit point of $(f,\phi)(H,A)$.
Therefore, $(f,\phi)(H,A)$ is a dense in $(\tilde{Y},\tilde{d}_2)$.

Theorem 2.4. Every soft isometry map is a soft continuous map.

Proof: Let $(f,\phi):(\tilde{X},\tilde{d}_1,E_1)\to(\tilde{Y},\tilde{d}_2,E_2)$ is a soft isometry.
$\Rightarrow\tilde{d}_2((f,\phi)P_\lambda^{\ x},(f,\phi)P_\mu^{\ y}))=\tilde{d}_1(P_\lambda^{\ x},P_\mu^{\ y}).$
Now take $\tilde{\varepsilon}=\tilde{d}_1(P_\lambda^{\ x},P_\mu^{\ y})=\tilde{\delta}.$
$\Rightarrow\tilde{d}_2((f,\phi)(P_\lambda^{\ x}),(f,\phi)(P_\mu^{\ y}))\tilde{<}\tilde{\varepsilon}$ whenever
$\tilde{d}_1(P_\lambda^{\ x},P_\mu^{\ y})\tilde{<}\tilde{\delta}.$
Hence, (f,ϕ) is soft continuous.

Remark 2.5. From the proof of the theorem2.5, we can see that**,** every soft isometry map is soft uniformly continuous.

III. Soft Isometry

Definition 3.1 Let $(\tilde{X},\tilde{d})$ be a metric space we say that $(\tilde{X}^*,\tilde{d}^*)$ is a soft completion of $(\tilde{X},\tilde{d})$ if

1. $(\tilde{X}^*,\tilde{d}^*)$ is a complete soft metric space.
2. $(\tilde{X},\tilde{d})$ is soft isometric to some dense subset of $(\tilde{X}^*,\tilde{d}^*)$ .i.e. there exist a soft isometry $(f,\phi):(\tilde{X},\tilde{d})\to(\tilde{X}^*,\tilde{d}^*)$ such that $f(\tilde{X})$ is dense in $\tilde{X}^*$.

Remark 3.2 Let $(X,\tilde{d},E)$ be a soft metric space then let
$B(\tilde{X})=\{(f,\phi):(X,\tilde{d},E)\to\mathbb{R}(E)\,|\,(f,\phi)\text{is soft bounded map}\}$
is the set of all soft bounded soft real valued map on $\tilde{X}$.

Define $\| f \| = \sup_{P_\lambda{}^x \in (X,E)} |(f,\phi)(P_\lambda{}^x)|$ then $B(\tilde{X})$ is soft complete with respect to $\| . \|$.

Lemma 3.3 Let $(X,\tilde{d},E)$ be a soft metric space then let then $\tilde{X}$ is soft isometric to a subset of $B(\tilde{X})$.

Proof: Fixed any soft point $P_\sigma{}^a \in (X,E)$ then for any $P_\lambda{}^x \in (X,E)$, define $(f,\phi)_{P_\lambda{}^x} : (X,E) \to \mathbb{R}(E)$ as $(f,\phi)_{P_\lambda{}^x}\ (P_\gamma{}^t) = \tilde{d}(P_\lambda{}^x, P_\gamma{}^t) - \tilde{d}(P_\sigma{}^a, P_\gamma{}^t)$ then

$$|(f,\phi)_{P_\lambda{}^x}\ (P_\gamma{}^t) = \tilde{d}(P_\lambda{}^x, P_\gamma{}^t) - \tilde{d}(P_\sigma{}^a, P_\gamma{}^t)|$$

$$\tilde{\leq} \tilde{d}(P_\lambda{}^x, P_\sigma{}^a) = \tilde{c}$$

$\therefore |(f,\phi)_{P_\lambda{}^x}\ (P_\gamma{}^t)| \tilde{\leq} \tilde{c}\ \ \forall P_\gamma{}^t \tilde{\in} \tilde{X}$.

$\Rightarrow (f,\phi)_{P_\lambda{}^x} : (X,E) \to \mathbb{R}(E)$ is soft bounded.

$\Rightarrow (f,\phi)_{P_\lambda{}^x} \in B(\tilde{X})$.

Now define, $(h,\psi):(X,E) \to S(\tilde{X}) \subset B(\tilde{X})$

Such that $(h,\psi)(P_\lambda{}^x) = (f,\phi)_{P_\lambda{}^x}$ where

$$S(\tilde{X}) = \{(f,\phi)_{P_\lambda{}^x} : (X,E) \to \mathbb{R}(E) \text{is soft bounded.} | P_\lambda{}^x \tilde{\in} \tilde{X}\}$$

For any $P_\lambda{}^x, P_\mu{}^y \in \tilde{X}$,

$$\| (f,\phi)_{P_\lambda{}^x} - (f,\phi)_{P_\mu{}^y} \| = |(f,\phi)_{P_\lambda{}^x}(P_\gamma{}^t) - (f,\phi)_{P_\mu{}^y}(P_\gamma{}^t)|$$

$$|(f,\phi)_{P_\lambda{}^x}(P_\gamma{}^t) - (f,\phi)_{P_\mu{}^y}(P_\gamma{}^t)|$$

$$= |\tilde{d}(P_\lambda{}^x, P_\gamma{}^t) - \tilde{d}(P_\sigma{}^a, P_\gamma{}^t) - \tilde{d}(P_\mu{}^y, P_\gamma{}^t) + \tilde{d}(P_\sigma{}^a, P_\gamma{}^t)|$$

$$= |\tilde{d}(P_\lambda{}^x, P_\gamma{}^t) - \tilde{d}(P_\mu{}^y, P_\gamma{}^t)|$$

$$\leq \tilde{d}(P_\lambda{}^x, P_\mu{}^y).$$

For $P_\gamma{}^t = P_\lambda{}^x$, $|(f,\phi)_{P_\lambda{}^x}(P_\gamma{}^t) - (f,\phi)_{P_\mu{}^y}(P_\gamma{}^t)| = \tilde{d}(P_\lambda{}^x, P_\mu{}^y)$

$$\sup_{P_\gamma{}^t \in \tilde{X}} |(f,\phi)_{P_\lambda{}^x}(P_\gamma{}^t) - (f,\phi)_{P_\mu{}^y}(P_\gamma{}^t)| = \tilde{d}(P_\lambda{}^x, P_\mu{}^y)$$

$$\Rightarrow \| (f,\phi)_{P_\lambda{}^x}(P_\gamma{}^t) - (f,\phi)_{P_\mu{}^y}(P_\gamma{}^t) \| = \tilde{d}(P_\lambda{}^x, P_\mu{}^y)$$

$$\Rightarrow \| (h,\psi)_{P_\lambda{}^x} - (h,\psi)_{P_\mu{}^y} \| = \tilde{d}(P_\lambda{}^x, P_\mu{}^y).$$

$\Rightarrow (h,\psi)$ is a soft isometry from $\tilde{X}$ to $S(\tilde{X}) \subset B(\tilde{X})$.

Therefore, $\tilde{X}$ is a soft isometric to subset of $B(\tilde{X})$.

Theorem 3.4 (Completion a Soft Metric Space)

Every soft metric space has a completion.

Proof:

$$S(\tilde{X}) = \{(f,\phi)_{P_\lambda{}^x} : (X,E) \to \mathbb{R}(E) \text{is soft bounded} | P_\lambda{}^x \tilde{\in} \tilde{X}\}$$

is a soft subset of $B(\tilde{X})$. Then $\overline{S(\tilde{X})}$ îs a soft closed subset of $B(\tilde{X})$ and hence $\overline{S(\tilde{X})}$ is soft complete.

$(h,\psi):(X,E) \to S(\tilde{X})$ defined as

$(h,\psi)(P_\lambda{}^x) = (f,\phi)_{P_\lambda{}^x}$ is a soft isometry.

Then $(h,\psi)(\tilde{X}) = S(\tilde{X})$ and $\overline{(h,\psi)(\tilde{X})} = \overline{S(\tilde{X})}$ ^

$\Rightarrow (h,\psi)(\tilde{X})$ is dense in $S(\tilde{X})$.

Hence $\overline{S(\tilde{X})}$ is a soft completion of $\tilde{X}$.

Theorem 3.5 If $\tilde{X}_1$ and $\tilde{X}_2$ are completions of $\tilde{X}$, then $\tilde{X}_1$ and $\tilde{X}_2$ are soft isometric.

Proof: Since $\tilde{X}_1$ is a soft completion of $\tilde{X}$, there exist

$(f,\phi):(X,\tilde{d},E) \to (X_1,\tilde{d}_1 E_1)$ is soft isometry such

that $(f,\phi)((X,E))$ is dense in (X_1,E_1).

Since $\tilde{X}_2$ is a soft completion of $\tilde{X}$, there exist

$(g,\psi):(X,\tilde{d},E) \to (X_2,\tilde{d}_2 E_2)$ is soft isometry such

that $(g,\psi)((X,E))$ is dense in (X_2,E_2).

Let $P_\lambda{}^x \in (X_1,E_1)$ .Since $(f,\phi)((X,E))$ is dense in (X_1,E_1) there exists $(P_{\lambda_n}{}^{x_n})$ in (X,E) such that $(f,\phi)(P_{\lambda_n}{}^{x_n}) \to P_\lambda{}^x$ then we will show that $(g,\psi)((P_{\lambda_n}{}^{x_n}))$ is Cauchy in (X_2,E_2).

$$\tilde{d}_2((g,\psi)(P_{\lambda_n}{}^{x_n}),(g,\psi)(P_{\mu_m}{}^{y_m})) = \tilde{d}(P_{\lambda_n}{}^{x_n}, P_{\mu_m}{}^{y_m})$$

$$= \tilde{d}_1((f,\phi)P_{\lambda_n}{}^{x_n}, (f,\phi)P_{\mu_m}{}^{y_m})$$

Since $((f,\phi)(P_{\lambda_n}{}^{x_n}))$ is Cauchy, $((g,\psi)(P_{\lambda_n}{}^{x_n}))$ is also Cauchy. Since (X_2,E_2) is soft complete

$((g,\psi)(P_{\lambda_n}{}^{x_n}))$ converges in $(X_2,\tilde{d}_2)$ say $'P_\mu{}^y{}'$ i.e.

$(g,\psi)(P_{\lambda_n}{}^{x_n}) \to P_\mu{}^y$, $P_\mu{}^y \tilde{\in} (X_2,\tilde{d}_2)$.

Now define, $(h,\varphi):(X_1,E_1,\tilde{d}_1) \to (X_2,E_2,\tilde{d}_2)$ as

$(h,\varphi)P_\lambda{}^x = P_\mu{}^y$.

Now we will show that (h,φ) is a soft isometry.

Let $P_{\lambda_1}{}^{x_1}, P_{\lambda_2}{}^{x_2} \in (X_1,\tilde{d}_1)$ then there exists

$P_{\lambda_n}{}^{x_n} P_{\lambda_n'}{}^{x_n'} \in (X,E)$ and $P_{\mu_1}{}^{y_1}, P_{\mu_2}{}^{y_2} \in (X_2,\tilde{d}_2)$ such that

$(h,\varphi)(P_{\lambda_1}{}^{x_1}) = P_{\mu_1}{}^{y_1}$ and $(h,\varphi)(P_{\lambda_2}{}^{x_2}) = P_{\mu_2}{}^{y_2}$

And $(f,\phi)(P_{\lambda_n}{}^{x_n}) \to P_{\lambda_1}{}^{x_1}$, $(f,\phi)(P_{\lambda_n'}{}^{x_n'}) \to P_{\lambda_2}{}^{x_2}$

$(g,\psi)(P_{\lambda_n}{}^{x_n}) \to P_{\mu_1}{}^{y_1}$, $(g,\psi)(P_{\lambda_n'}{}^{x_n'}) \to P_{\mu_2}{}^{y_2}$.

Since $(f,\phi)(P_{\lambda_n}{}^{x_n}) \xrightarrow{\tilde{d}_1} P_{\lambda_1}{}^{x_1}$, $(f,\phi)(P_{\lambda_n'}{}^{x_n'}) \to P_{\lambda_2}{}^{x_2}$

$$\Rightarrow \tilde{d}_1((f,\phi)(P_{\lambda_n}{}^{x_n}),(f,\phi)(P_{\lambda_n'}{}^{x_n'})) \to \tilde{d}_1(P_{\lambda_1}{}^{x_1},P_{\lambda_2}{}^{x_2})$$

$$\Rightarrow \tilde{d}_1(P_{\lambda_1}{}^{x_1},P_{\lambda_2}{}^{x_2}) = \lim_{n\to\infty} \tilde{d}_1((f,\phi)(P_{\lambda_n}{}^{x_n}),(f,\phi)(P_{\lambda_n'}{}^{x_n'}))$$

$$= \lim_{n\to\infty} \tilde{d}\ (P_{\lambda_n}{}^{x_n},P_{\lambda_n'}{}^{x_n'})$$

$$= \lim_{n\to\infty} \tilde{d}_2((g,\psi)(P_{\lambda_n}{}^{x_n}),(g,\psi)(P_{\lambda_n'}{}^{x_n'}))$$

$$= \tilde{d}_2((P_{\mu_1}{}^{y_1}),(P_{\mu_2}{}^{y_2}))$$

$$\{\because (g,\psi)(P_{\lambda_n}{}^{x_n}) \to P_{\mu_1}{}^{y_1},(g,\psi)(P_{\lambda_n'}{}^{x_n'}) \to P_{\mu_2}{}^{y_2}$$

$$\Rightarrow \tilde{d}_2((g,\psi)(P_{\lambda_n}{}^{x_n}),(g,\psi)(P_{\lambda_n'}{}^{x_n'})) \to \tilde{d}_2(P_{\mu_1}{}^{y_1},P_{\mu_2}{}^{y_2})\}$$

$$= \tilde{d}_2((h,\psi)(P_{\lambda_1}{}^{x_1}),(h,\psi)(P_{\lambda_2}{}^{x_2}))$$

$\Rightarrow (h,\varphi)$ is a soft isometry.

Therefore, (X_1,E_1) and (X_2,E_2) are soft isometric.

Conclusions

In this paper, we have defined soft isometric and derived some results on soft isometry. We also defined completion of a soft metric space and proved that every soft metric space has a unique completion up to isometry.

One of the authors is thankful to UGC for providing financial support thorough MAULANA AZAD NATIONAL FELLOWHIP.

Acknowledgment

One of the authors is thankful to UGC for providing financial support thorough MAULANA AZAD NATIONAL FELLOWHIP.

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